Recitation 2

1. Exercise 1.22: Find the coefficient of $x^3y^2z^3w$ in the expansion of $(2x + 3y - 4z + w)^9$.
2. How many terms are there in the expansion given in the previous exercise?
3. Exercise 2.4: Use Venn diagrams to verify that
   (a) $(A \cap B) \cup (A \cap B^C) = A$;
   (b) $A \cap (B \cup C)$ is the same event as $(A \cap B) \cup (A \cap C)$;
   (c) $A \cup (B \cap C)$ is the same event as $(A \cup B) \cap (A \cup C)$.
4. Let E, F, G be three events. Find expressions for the events that of E, F, G
   (a) only F occurs,
   (b) both E and F but not G occur,
   (c) at least one event occurs,
   (d) at least two events occur,
   (e) all three events occur,
   (f) none occurs,
   (g) at most one occurs,
   (h) at most two occur.
5. Exercise 2.41: A coin is tossed once. Then, if it comes up heads, a die is thrown once; if the coin comes up tails, it is tossed twice more. Using the notation in which $(H, 2)$, for example denotes the event that the coin comes up heads and then the die comes up 2, and (T, T, T) denotes the event that the coin comes up tails three times in a row, list
   (a) the 10 elements of the sample space;
   (b) the elements of the sample space corresponding to event A that exactly one head occurs;
   (c) the elements of the sample space corresponding to event B that at least two tails occur or a number of greater than 4 occurs.
6. In a lot of practical examples (think of a postoffice or a call center) we want to set up a random experiment measuring the number of customers in an arrival process. In this random experiment we like to determine the number of arriving customers within the interval $[0, t]$ for every $t > 0$. What is a possible outcome or elementary event in this random experiment? Draw a possible realization.
7. Consider a postoffice with only one server and a random arrival process of single arriving customers. Every customer requires a random service time and the queueing discipline is fist-come first-served. Customers who have to wait are waiting in a queue. The random experiment is to observe the total number of customers in the system at any time in the interval $[0, \infty)$. What is a possible outcome or elementary event in this random experiment? Draw a possible realization.

Use the following definition of independence for the remaining problems.

- Two events A, B are *independent* if and only if $P(A \cap B) = P(A)P(B)$.
- n events $A_1, A_2, \ldots, A_n$ are *independent* if

$$P(A_{i_1} \cap \ldots A_{i_r}) = P(A_{i_1})P(A_{i_2}) \ldots P(A_{i_r})$$

  holds for any subcollection $i_1, i_2, \ldots, i_r$ of numbers $\{1, 2, \ldots, n\}$ for $2 \leq r \leq n$.

8. In a random experiment, it is given that

$$\mathbb{P}(A \cap B') = 0.47, \quad \mathbb{P}(A' \cap B') = 0.1, \quad \mathbb{P}(A' \cup B') = 0.84.$$

Find

   (a) $\mathbb{P}(A)$;
   (b) $\mathbb{P}(A \cup B)$;
   (c) $\mathbb{P}(B)$.

9. If A and B are independent events, prove that A and B' are also independent. That is, prove the equality $\mathbb{P}(A \cap B') = \mathbb{P}(A) \cdot \mathbb{P}(B')$ using the equality $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$.

10. List the conditions that are necessary to check that events A, B, C are independent.

11. Exercise 2.30: Show that $2^k - k - 1$ conditions must be satisfied for k events to be independent.